International Journal of Theoretical Physics, Vol. 38, No. 5, 1999

Dynamical Model and Path Integral Formalism for Hubbard Operators

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Received April 28, 1998

In this paper, the possibility to construct a path integral formalism by using the Hubbard operators as field dynamical variables is investigated. By means of arguments coming from the Faddeev–Jackiw symplectic Lagrangian formalism as well as from the Hamiltonian Dirac method, it can be shown that it is not possible to define a classical dynamics consistent with the full algebra of the Hubbard X-operators. Moreover, from the Faddeev–Jackiw symplectic algorithm, and in order to satisfy the Hubbard X-operators commutation rules, it is possible to determine the number of constraints that must be included in a classical dynamical model. Following this approach, it is clear how the constraint conditions that must be introduced in the classical Lagrangian formulation are weaker than the constraint conditions imposed by the full Hubbard operators algebra. The consequence of this fact is analyzed in the context of the path integral formalism. Finally, in the framework of the perturbative theory, the diagrammatic and the Feynman rules of the model are discussed.

1. INTRODUCTION

The Hubbard X-operators[(1)] are suitable to give a powerful framework in which the elementary excitations in solids can be explained. The use of X-operators is also relevant when electronic correlations are taken into account. This is the scenery in which high-T_c superconductivity occurs, and so the main reason why the Hubbard operator algebra is so interesting at the present time.

The algebra of the Hubbard $\hat{X}$-operators is completely defined by:

(a) The commutation rules

$$[\hat{X}_i^{\alpha\beta}, \hat{X}_j^{\gamma\delta}] = \delta_{ij}(\delta^{\beta\gamma}\hat{X}_i^{\alpha\delta} - \delta^{\alpha\delta}\hat{X}_i^{\gamma\beta}) \tag{1.1}$$

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0020-7748/99/0500-143 9$16.00/0

(b) The completeness condition

$$\hat{X}_i^{++} + \hat{X}_i^{--} = \hat{I} \tag{1.2}$$

(c) The multiplication rules for a given site

$$\hat{X}_i^{\alpha\beta}\, \hat{X}_i^{\gamma\delta} = \delta^{\beta\gamma}\, \hat{X}_i^{\alpha\delta} \tag{1.3}$$

From now on and for simplicity we consider the case in which the indices α, β can only take the values $+$ and $-$, and so the Hubbard $\hat{X}$-operators are boson-like operators of the $SU(2)$ algebra. The spin $s = 1/2$ is naturally contained in this case.

It is easy to show that equations (1.3) are not all independent, and so the full information contained in the algebra can be recovered from equations (1.1) and (1.2), and the following three independent equations:

$$\hat{X}^{-+}\, \hat{X}^{++} - \hat{X}^{-+} = 0 \tag{1.4a}$$

$$\hat{X}^{+-}\, \hat{X}^{--} - \hat{X}^{+-} = 0 \tag{1.4b}$$

$$\hat{X}^{+-}\, \hat{X}^{-+} - \hat{X}^{++} = 0 \tag{1.4c}$$

Consequently, the full algebra given by equations (1.1)–(1.3) is equivalent to the commutation rules (1.1), the completeness condition (1.2), and the three conditions (1.4).

A many-body theory constructed by using the Hubbard operators as field variables requires the application of techniques used in quantum field theories. From this point of view it is necessary to formulate the Wick theorem for the case in which the field operators are neither usual fermions nor bosons. Progress in this direction has been made,[(2)] but the problem is still open.

As in quantum field theories, another way to attack the problem is via the path integral formulation. It is important to note that a suitable path integral formulation must be independent of a given representation. On the other hand, it must be written in terms of an effective action with a well-defined dynamics. This last point of view will be adopted in the present paper.

The paper is organized as follows. In Sections 2 and 3 by using the Faddeev–Jackiw (FJ) Lagrangian method,[(3)] a general treatment for first-order Lagrangian systems containing the Hubbard operators as dynamical variables is given. A family of Lagrangians describing these dynamical systems is found. The use of these classical Lagrangians in a path integral quantization formalism is also analyzed. Strong arguments can be given showing that it is not possible to include the full Hubbard algebra (1.1)–(1.3) in a classical dynamical model. In Section 4 we confront our results with others previously given in the literature. In Section 5 the diagrammatic and the Feynman rules for the model are constructed. Finally, conclusions and discussions are given in Section 6.

2. CLASSICAL LAGRANGIAN AND DYNAMICAL MODEL

One of the traditional approaches to studying the quantization of spin systems or the t-J model in which the Hubbard operator algebra takes place is to consider the constrained systems from the point of view of coherent state phase path integration. Also frequently used is the usual Dirac Hamiltonian method for constrained systems by considering the slave boson or fermion representation.

By writing a family of first-order classical Lagrangians directly in terms of the four Hubbard operators, our main purpose is to obtain information about the kind and the number of constraints present in these models. In this way it is possible to find how much of the information contained in the algebra (1.1)–(1.3) can be introduced at the classical level. This approach requires the introduction of a suitable set of constraints, *a priori* unknown, that must be determined later on. To this purpose it is useful to use the FJ Lagrangian method.[3–6] Therefore, we briefly indroduce some definitions and key equations.

As is well known, the FJ symplectic quantization method is formulated on actions only containing first-order time derivatives. The most general first-order Lagrangian is specified in terms of two arbitrary functionals $K_A(\mu^A)$ and $\mathbf{V}^{(0)}(\mu)$, and is given by

$$L(\mu_A, \dot{\mu}^A) = \dot{\mu}^A K_A(\mu^A) - \mathbf{V}^{(0)}(\mu) \tag{2.1}$$

The functionals $K_A(\mu^A)$ are the components of the canonical one-form $K(\mu) = K_A(\mu)d\mu^A$, and the functional $\mathbf{V}^{(0)}(\mu)$ is the symplectic potential. The general compound index A runs over the different ranges of the complete set of variables that defines the extended configuration space.

The Euler–Lagrange equations of motion obtained from (2.1) are

$$\sum_B M_{AB}\dot{\mu}^B - \frac{\partial \mathbf{V}^{(0)}}{\partial \mu^A} = 0 \tag{2.2}$$

The elements of the symplectic matrix $M_{AB}(\mu)$ are the components of the symplectic two-form $M(\mu) = dK(\mu)$. The exterior derivative of the canonical one-form $K(\mu)$ is written as the generalized curl constructed with partial derivatives and so the components are given by

$$M_{AB} = \frac{\partial K_B}{\partial \mu^A} - \frac{\partial K_A}{\partial \mu^B} \tag{2.3}$$

When the symplectic matrix M_{AB} is nonsingular, we obtain from the equations of motion (2.2)

$$\dot{\mu}^A = (M^{AB})^{-1} \frac{\partial \mathbf{V}^{(0)}}{\partial \mu^B} \tag{2.4}$$

As the symplectic potential is just the Hamiltonian of the system, equation (2.4) is written

$$\dot{\mu}^A = [\mu^A, \mathbf{V}] = [\mu^A, \mu^B] \frac{\partial \mathbf{V}^{(0)}}{\partial \mu^B} \tag{2.5}$$

where

$$[\mu^A, \mu^B] = (M^{AB})^{-1} \tag{2.6}$$

are the generalized brackets defined in the FJ symplectic formalism.

It is easy to show that the elements $(M^{AB})^{-1}$ of the inverse of the symplectic matrix M_{AB} correspond to the Dirac brackets[7] of the theory.

Transition to the quantum theory is realized as usual by replacing classical fields by quantum field operators acting on the Hilbert space, where quantum ordering and proper boundary conditions for the quantum field operators must be taken into account. Therefore, the predictions of the FJ and Dirac methods are equivalent.

When the matrix M^{AB} is singular, the constraints appear as algebraic relations and they are necessary to maintain the consistency of the field equations of motion. In such a case, there exist m ($m < n$) left (or right) zero modes $\mathbf{v}_a$ ($a = 1, \ldots, m$, $A = 1, \ldots, n$) of the supermatrix M_{AB}, where each $\mathbf{v}_a$ is a column vector with $n + m$ entries v_a^A. So the zero modes satisfy

$$\sum_A v_a^A M_{AB} = 0 \tag{2.7}$$

From the equations of motion (2.2) we see that the quantities Ω_a are the true constraints in the FJ symplectic formalism, and they are given by

$$\Omega_a = v_a^i \frac{\partial}{\partial \varphi^i} \mathbf{V}^{(0)} = 0 \tag{2.8}$$

Consequently, in a first iteration the constraints are written in the symplectic part of the Lagrangian by means of Lagrange multipliers as follows:

$$L^{(1)} = \dot{\varphi}^i a_i(\varphi) + \dot{\xi}^a \Omega_a - \mathbf{V}^{(1)} \tag{2.9}$$

where the new symplectic potential is by definition $\mathbf{V}^{(1)} = \mathbf{V}^{(0)}\big|_{\Omega=0}$. The partition $\mu^A = (\varphi^i, \xi^a)$ and $K_A = (a_i, \Omega_a)$ has been made. So, the compound indices A, B run over the sets $A = (i, a)$ and $B = (j, b)$.

In each iterative procedure the configuration space is enlarged and the symplectic matrix is modified. When no new constraints are found the iterative procedure is finished.

Now we apply the FJ quantization formalism to a dynamical model for the Hubbard operators.

As is well known, in all the examples in which the field variables are the components of the spin operators, the starting point is to consider first-order Lagrangians. This also happens in the t-J model when it is written in slave boson or fermion representation.[8] The FJ quantization algorithm is suitable to study this kind of dynamical system described by constrained first-order Lagrangians in which the constraints play a crucial role.

Therefore, in the case under consideration we assume that the first-order classical Lagrangian as functional of the Hubbard operators is written as follows:

$$L = a_{\alpha\beta}(X)\dot{X}^{\alpha\beta} - H(X) - \lambda_a\Omega^a \tag{2.10}$$

where $H(X)$ is, for instance, the Hamiltonian of the Heisenberg model written in terms of the Hubbard operators. The site indices were dropped since they are irrelevant in the analysis we will develop. The site indices can be included without any difficulty.

In equation (2.10) λ_a is an adequate set of Lagrange multipliers which allows the introduction of the constraints in the Lagrangian formalism. $\Omega^a(X)$ is a set of suitable unknown constraints, initially considered ad hoc in the Lagrangian. Both the constraints $\Omega^a(X)$ as well as the range of the index a must be determined by consistency. The coefficients $a_{\alpha\beta}(X) = a^*_{\beta\alpha}(X)$ are found in such a way that the algebra (1.1)–(1.3) for the Hubbard operators must be satisfied.

Looking at equation (2.10), we see that the initial set of dynamical symplectic variables is defined by $(X^{\alpha\beta}, \lambda_a)$ and the symplectic potential $\mathbf{V}^{(0)}$ is given by

$$\mathbf{V}^{(0)} = H(X) + \lambda_a\Omega^a \tag{2.11}$$

So, the symplectic matrix (2.3) obtained from the Lagrangian (2.10) is singular; therefore the constraints are obtained by using equation (2.8) and they read

$$\frac{\partial \mathbf{V}^{(0)}}{\partial \lambda_a} = \Omega^a \tag{2.12}$$

and the first-iterated Lagrangian reads

$$L^{(1)} = a_{\alpha\beta}(X)\dot{X}^{\alpha\beta} + \dot{\xi}_a\Omega^a - H(X) \tag{2.13}$$

The modified symplectic matrix associated to the Lagrangian (2.13) is

$$M_{AB} = \begin{pmatrix} \dfrac{\partial a_{\gamma\delta}}{\partial X^{\alpha\beta}} - \dfrac{\partial a_{\alpha\beta}}{\partial X^{\gamma\delta}} & \dfrac{\partial \Omega_b}{\partial X^{\alpha\beta}} \\ -\dfrac{\partial \Omega_a}{\partial X^{\gamma\delta}} & 0 \end{pmatrix} \tag{2.14}$$

with the indices $A = \{(\alpha\beta), a\}$, $B = \{(\gamma\delta), b\}$.

At this stage the problem is to determine which and how many constraints can be deduced from the algorithm of the method in such a way as to obtain a nonsingular symplectic matrix.

In this way, from the Lagrangian (2.13) the symplectic matric (2.14) is constructed and its inverse can be computed. By equating each of the elements $(M^{AB})^{-1}$ of the inverse of the symplectic matrix M_{AB} to each one of the commutation rules (1.1), differential equations on the coefficients $a_{\alpha\beta}(X)$ and on the constraints Ω^a are obtained.

As can be seen, the dimension of the symplectic matrix (2.14) is $4 + a$, where a enumerates the constraints. Because of the antisymmetric property of M_{AB} the index a has even range. From the properties of this matrix we can conclude:

(i) If $a > 4$ or odd, the symplectic matrix is singular.

(ii) For $a = 4$ the symplectic matrix can be invertible, but it is not possible to obtain the commutation rules (1.1). The commutators obtained by using equation (2.6) vanish, independently of the value of the coefficients $a_{\alpha\beta}(X)$. On the other hand, when the number of constraints equals the number of fields there is no dynamics. So it is not possible by means of Lagrange multipliers to enforce the constraint (1.2) together with the other three conditions (1.4).

Consequently, we cannot introduce in a classical first-order Lagrangian the complete information contained in the algebra (1.1)–(1.3).

Then, the unique possibility is to have only two constraints. Equation (1.2) or the completeness condition must be imposed to account for their physical meaning. This avoids at the quantum level the configuration with doubly occupied sites. The remaining constraint cannot be any of those given in (1.4), because the commutators (1.1) cannot be recovered. Therefore, we can expect that the remaining constraint can be provided naturally by consistency, when the symplectic method is used.

Consequently, we assume an arbitrary constraint $\Omega = \Omega\,(X^{+-}, X^{-+}, u)$, where $u = X^{++} - X^{--}$. This assumption is not a restriction because, by the completeness condition, the sum $(X^{++} + X^{--})$ is equal to one. From the requirement that the matrix elements of the inverse of the symplectic matrix (2.14) must be equal to each of the Hubbard commutation rules (1.1), and by solving the differential equation on this constraint the solution, we find

$$\Omega = X^{+-} X^{-+} + \tfrac{1}{4} u^2 - \beta = 0 \tag{2.15}$$

where β is an arbitrary constant.

We emphasize that the constraint (2.15) is not an imposition but appears naturally from our method. This is the unique possible constraint in order to satisfy the commutation rules and the completeness condition. Of course in equation (2.15) there is less dynamical information than that contained in the three equations (1.4).

We will discuss this point in relation to the fact that the path integral for this kind of field represents the system in some limit of the operatorial approach.

3. DETERMINATION OF THE LAGRANGIAN COEFFICIENTS

The next step is to determine the functions $a_{\alpha\beta}(X)$ written in the Lagrangian (2.13). The two constraints Ω_a we must consider are given in equations (1.2) and (2.15). Once the symplectic matrix (2.14) is constructed, its inverse can be computed. Taking into account the equation (2.6) and the commutation rules (1.1), by consistency we find the following differential equation:

$$2\left[\frac{\partial a_{+-}}{\partial u} - \frac{\partial a_u}{\partial X^{+-}}\right]X^{+-} - 2\left[\frac{\partial a_{-+}}{\partial u} - \frac{\partial a_u}{\partial X^{-+}}\right]X^{-+} + \left[\frac{\partial a_{-+}}{\partial X^{+-}} - \frac{\partial a_{+-}}{\partial X^{-+}}\right]u = i \tag{3.1}$$

where $a_u = \frac{1}{2}(a^{++} - a^{--})$.

We assume that the coefficients a_{+-}, a_{-+}, and a_u can be written as products of arbitrary functions of the u variable by polynomials in the X^{+-} and X^{-+} variables. For simplicity we look for a particular family of solutions by taking first-order polynomials in the X^{+-} and X^{-+} variables, i.e.,

$$a_{+-} = f(u)[e + bX^{+-} + cX^{-+}] \tag{3.2a}$$

$$a_{-+} = a^*_{+-} = f^*(u)[e^* + c^*X^{+-} + b^*X^{-+}] \tag{3.2b}$$

$$a_u = h(u)[p + qX^{+-} + rX^{-+}] \tag{3.2c}$$

where the constant coefficients p, q, r, e, b, and c are arbitrary.

Once the expressions (3.2) are introduced in equation (3.1), we find by straightforward computation

$$ph(u) = (ph(u))^* \tag{3.3a}$$

$$qh(u) = (rh(u))^* \tag{3.3b}$$

$$qh(u) = e \frac{df}{du} \tag{3.3c}$$

$$cf(u) - c^*f^*(u) = 2i \text{ Im } cf = 2i \frac{u + \alpha}{4\beta - u^2} \tag{3.3d}$$

with the conditions $b = 0$, and α an arbitrary integration constant.

Consequently, the equations (3.2) for the Lagrangian coefficients and (3.3) determine a family of Lagrangians compatible with the commutation rules (1.1), the completeness condition (1.2), and the constraint (2.15).

Not losing generality, in equations (3.2b) and (3.3d) we can choose $c = i$, finding the function $f(u)$

$$f(u) = \frac{u + \alpha}{4\beta - u^2}$$

and so two different families of solutions are obtained:

(i) If $e = 0$, the solution reads

$$a_{+-} = i \frac{u + \alpha}{4\beta - u^2} X^{-+} \tag{3.4a}$$

$$a_{-+} = -i \frac{u + \alpha}{4\beta - u^2} X^{+-} \tag{3.4b}$$

$$a_u = \frac{1}{2} (a_{++} - a_{--}) = h(u) \tag{3.4c}$$

where $h(u)$ is an arbitrary real function which also can be taken equal to zero.

(ii) If $e \neq 0$, the solution reads

$$a_{+-} = \frac{u + \alpha}{4\beta - u^2} (1 + iX^{-+}) \tag{3.5a}$$

$$a_{-+} = \frac{u + \alpha}{4\beta - u^2} (1 - iX^{+-}) \tag{3.5b}$$

$$a_u = \frac{1}{2} (a_{++} - a_{--}) = h(u) [1 + X^{+-} + X^{-+}] \tag{3.5c}$$

where in this second case $h(u)$ satisfies equation (3.3c).

The two different families of solutions (3.4) and (3.5) take into account the majority of the significant cases.

Finally, we make the following linear transformation to real variables (S_1, S_2, S_3),

$$X^{+-} = S_1 + iS_2 \tag{3.6a}$$

$$X^{-+} = S_1 - iS_2 \tag{3.6b}$$

$$X^{++} - X^{--} = 2S_3 \tag{3.6c}$$

and define the vectors $\mathbf{a} = (a_{S_1}, a_{S_2}, a_{S_3})$, $\nabla = (\partial_{S_1}, \partial_{S_2}, \partial_{S_3})$, and $\mathbf{S} = (S_1, S_2, S_3)$, where

$$a_{S_1} = a_{+-} + a_{-+} \tag{3.7a}$$

$$a_{S_2} = i(a_{+-} - a_{-+}) \tag{3.7b}$$

$$a_{S_3} = a_{++} - a_{--} \tag{3.7c}$$

Thus we can write equation (3.1) in the following simpler way:

$$(\nabla \times \mathbf{a})\boldsymbol{\cdot}\mathbf{S} = 1 \tag{3.8}$$

The form of the differential equation (3.8) is equal to that obtained in refs. 9 and 10. Then, the fact that the kinetic term can be written as a function of a vector field **a** which satisfies equation (3.8) is recovered. Note that equation (3.8) is a good definition for a curl on an S^2 manifold. Then, equation (3.8) together with (2.15) written in terms of the new variables S_1, S_2, and S_3 allows us to write the kinetic term in the Lagrangian as the area of a sphere with radius $\beta^{1/2}$. This is the principal argument for saying that $\beta^{1/2}$ must be integer or half-integer. For a complete discussion about this argument see refs. 9 and 10.

4. A SIMPLE CASE AND ITS RELATION TO PREVIOUS WORK

From Section 3 we can assert that a large family of Lagrangian exists any one of which can be considered as a good candidate for describing the dynamics contained in the commutation rules of the X-operators. The aim of this section is to discuss some important points by using explicitly one of the possible Lagrangians found in the previous section. Thus, by taking $a_{++} = a_{--} = 0$ in equation (3.4c) and calling $\alpha = -2s$ and $\beta = s^2$, we can write the Lagrangian (2.10) as

$$L(X, \dot{X}) = \frac{i}{2}\left(\frac{X^{-+}\dot{X}^{+-} - X^{+-}\dot{X}^{-+}}{s + \frac{1}{2}(X^{++} - X^{--})}\right) - H(X) \tag{4.1}$$

with the two constraints

$$X^{+-}X^{-+} + \frac{1}{4}(X^{++} - X^{--})^2 = s^2 \tag{4.2a}$$

$$X^{++} + X^{--} = 1 \tag{4.2b}$$

Equations (4.1) and (4.2) describe the classical dynamics of a system in which the commutation rules (1.1) are satisfied.

We note that the same result also can be found by using the Dirac theory for constrained systems.[7] From this approach it is easy to show that the constraints given in equations (4.2) together with the constraints coming from the definition of the momentum of the X variables is a set of second-class constraints. The Dirac brackets associated to this set of constraints are exactly the correct commutation rules for the Hubbard operators.

Now we are able to write the following partition function by using the Faddeev–Senjanovic path integral approach[11]:

$$Z = \int DX\, \delta\left[X^{+-}\, X^{-+} + \frac{1}{4}(X^{++} - X^{--})^2 - s^2 \right] \delta(X^{++} + X^{--} - 1) \times \exp i \int dt\, L(X, \dot{X}) \tag{4.3}$$

where $L(X, \dot{X})$ is given by (4.1).

By integrating in the X^{--} variable we obtain the following expression for the partition function Z:

$$Z = \int DX^{-+}\, DX^{+-}\, DX^{++}\, \delta\left[X^{+-}\, X^{-+} + \frac{1}{4}(2X^{++} - 1)^2 \right] \times \exp i \int dt\, L^{(1)}(X, \dot{X}) \tag{4.4}$$

where

$$L^{(1)}(X, \dot{X}) = -\frac{i}{2} \frac{X^{+-}\dot{X}^{-+} - X^{-+}\dot{X}^{+-}}{s + \frac{1}{2}(2X^{++} - 1)} - H(X) \tag{4.5}$$

Making in equation (4.5) the change of variables

$$S_1 = \frac{X^{+-} + X^{-+}}{2} \tag{4.6a}$$

$$S_2 = \frac{X^{+-} - X^{-+}}{2i} \tag{4.6b}$$

$$S_3 = \frac{1}{2}(2X^{++} - 1) \tag{4.6c}$$

we can write the functional integral (4.4) as

$$Z = \int DS\, \delta(S_1^2 + S_2^2 + S_3^2 - s^2) \exp i \int dt\, L^{(2)}(S, \dot{S}) \tag{4.7}$$

where

$$L^{(2)}(S, \dot{S}) = \frac{S_2\dot{S}_1 - S_1\dot{S}_2}{s + S_3} - H(S) \tag{4.8}$$

where the constant Jacobian of the transformation (4.6) was absorbed in the functional integral measure. Therefore, equation (4.7) for the partition function agrees with the expression (3.17) of ref. 12, obtained by means of different arguments.

Now, it is easy to show that this expression is consistent with the quantization of a spin system in the limit of large s. Applying again the Dirac theory, but now to the Lagrangian (4.8) with the constraints

$$|\mathbf{S}|^2 = s^2 \tag{4.9}$$

we find that the second-class nature of the constraint defining the Dirac brackets is again exactly the commutation rules (1.1) for the spin components. It is interesting to note that in the quantization procedure, the second-class constraint (4.9) must be considered as a strong equation among operators. Then

$$\hat{S}^2 = s^2\hat{I} \tag{4.10}$$

From the comment given at the end of Section 3 it is known that the number s must be integer or half-integer. Consequently, it is not possible to write s^2 as s' $(s' + 1)$ with s' integer or half-integer in the equation (4.10).

This is an important reason that in the path integral formalism for the spin systems, the information of the large-s approximation is contained from the beginning. This fact is connected with our results making impossible the inclusion of the full X-operator algebra in a classical Lagrangian formalism, or equivalently in a path integral formulation.

5. DIAGRAMMATIC AND FEYNMAN RULES

Now, in order to obtain the diagrammatic and the Feynman rules for the model we analyze the perturbative treatment. The starting point is the following partition function:

$$Z = \int DX^{+-}\, DX^{-+}\, Du\, \delta\left(X^{+-}X^{-+} + \frac{1}{4}u^2 - \beta\right) \times \exp i \int dt\, L(X, \dot{X}) \tag{5.1}$$

where the integration on the $(X^{++} + X^{--})$ variable has been made by using the function $\delta(X^{++} + X^{--} - 1)$.

Consequently, the Lagrangian $L(X, \dot{X})$ can be written

$$L = a_{+-} \dot{X}^{+-} + a_{-+}\dot{X}^{-+} + a_{u}\dot{u} - H(X^{+-}, X^{-+}, u) \tag{5.2}$$

Taking into account equations (3.4) for the coefficients, we consider the perturbative development for large value of the parameter β. Therefore the nonpolynomial Lagrangian (5.2), up to first order in β^{-1}, reads

$$L(X, \dot{X}) = \frac{i\alpha}{4\beta}(X^{-+}\dot{X}^{+-} - X^{+-}\dot{X}^{-+}) + a_u\dot{u}$$
$$+ \frac{i}{4\beta}u(X^{-+}\dot{X}^{+-} - X^{+-}\dot{X}^{-+}) - H(X) \tag{5.3}$$

In equation (5.3), we consider for the Hamiltonian $H(X)$ the Heisenberg ferromagnetic form:

$$H(X) = -\frac{1}{2}J\left(X^{+-}X^{-+} + X^{-+}X^{+-} + \frac{1}{2}uu\right) \tag{5.4}$$

where $J > 0$.

By using in the path integral (5.1) the Gaussian representation

$$\delta(x) = \lim_{\sigma\to 0} \frac{1}{\pi\sqrt{\sigma}} \exp\left(-\frac{1}{\sigma}x^2\right)$$

for the delta function, we can write the partition function in terms of an effective Lagrangian as follows:

$$Z = \int DV \exp i \int_0^T dt\, L^{\text{eff}}(V) \tag{5.5}$$

In equation (5.5), the effective Lagrangian $L^{\text{eff}}(V)$ is written in terms of an extended complex vector field V whose components are given by

$$V = (X^{+-}, X^{-+}, u)$$

and it can be partitioned as follows:

$$L^{\text{eff}} = L^{(2)}(V) + L^{(3)}(V) + L^{(4)}(V) \tag{5.6}$$

As is usual the quadratic part $L^{(2)}(V)$ of the effective Lagrangian defines the free propagator of the model, and the remaining parts $L^{(3)}(V)$ and $L^{(4)}(V)$ represent the interaction vertices, i.e., the three- and four-leg vertices of the model, respectively. So, from equation (5.5) it can be seen that the quantum problem remains defined in terms of a path integral which contains the three independent fields X^{+-}, X^{-+}, and u.

In equation (5.6) the quadratic part $L^{(2)}(V)$ is given by

$$L^{(2)}(V) = \frac{1}{2} V^{\alpha} D_{\alpha\beta} V^{\beta} \tag{5.7}$$

where

$$D_{\alpha\beta} = \begin{pmatrix} 0 & \frac{i\alpha}{4\beta}\partial_t + \frac{\beta}{\sigma} + J & 0 \\ -\frac{i\alpha}{4\beta}\partial_t + \frac{\beta}{\sigma} + J & 0 & 0 \\ 0 & 0 & a\partial_t + \frac{\beta}{2\sigma} + \frac{J}{2} \end{pmatrix} \tag{5.8}$$

The simplest case in which $a_u = h(u) = au$ (where a is an arbitrary constant) was considered when the matrix (5.8) was computed.

The matrix $D_{\alpha\beta}$ is Hermitian and nondegenerate, and so the propagador $(D_{\alpha\beta})^{-1}$ in the $[q, \omega]$ space can be evaluated and we find

$$(D_{\alpha\beta})^{-1}(\omega, \omega')$$

$$= \begin{pmatrix} 0 & \frac{4\beta}{\alpha(\omega + 4\beta^2/\alpha\sigma - (4\beta/\alpha)J_q)} & 0 \\ \frac{4\beta}{\alpha(-\omega + 4\beta^2/\alpha\sigma - (4\beta/\alpha)J_q)} & 0 & 0 \\ 0 & 0 & \frac{1}{ia\omega + \beta/2\sigma - J_q} \end{pmatrix} \delta(\omega, \omega') \tag{5.9}$$

We note that J_q is the Fourier transform of $J_{ij} = J$ only if i, j are nearest neighbor sites.

The three- and four-leg vertices are respectively given by the parts

$$L^{(3)}(V) = \frac{1}{3!} \lambda_{\alpha\beta\gamma} V^{\alpha} V^{\beta} V^{\gamma} \tag{5.10}$$

$$L^{(4)}(V) = \frac{1}{4!} \lambda_{\alpha\beta\gamma\delta} V^{\alpha} V^{\beta} V^{\gamma} V^{\delta} \tag{5.11}$$

where

$$\lambda_{\alpha\beta\gamma} = \frac{1}{4\beta} (\omega' - \omega)\, \delta(-\omega + \omega' + \omega'')\delta(-q + q' + q''),$$
$$\alpha \neq \beta \neq \gamma \tag{5.12}$$

$$\lambda_{\alpha\beta\gamma\delta} = -\frac{3}{2\sigma}\,\delta(\omega + \omega' + \omega'' + \omega''')\delta(q + q' + q'' + q'''),$$
$$\alpha = \beta = \gamma = \delta = 3 \tag{5.13}$$

$$\lambda_{\alpha\beta\gamma\delta} = -\frac{1}{\sigma}\,\delta(-\omega + \omega' + \omega'' + \omega''')\delta(-q + q' + q'' + q'''),$$
$$\alpha = 1, \quad \beta = 2, \quad \gamma = \delta = 3$$
$$\text{and all the permutations} \tag{5.14}$$

$$\lambda_{\alpha\beta\gamma\delta} = -\frac{4}{\sigma}\,\delta(-\omega + \omega' - \omega'' + \omega''')\delta(-q + q' - q'' + q'''),$$
$$\alpha = 1, \quad \beta = 2, \quad \gamma = 1$$
$$\text{and all the permutations} \tag{5.15}$$

From the above results we can see that for $\alpha = -2\sqrt{\beta} = -2s$ and by choosing for the parameter σ the value $\sigma = \beta/Jz$, where z is the number of nearest neighbor sites, we obtain for the matrix element

$$(D_{12})^{-1} \equiv \langle T[X_q^{+-}(\omega)X_{q'}^{-+}(\omega')]\rangle = \frac{2s}{\omega - 2sz(J - J_q)}\,\delta(\omega - \omega')\delta(q - q') \tag{5.16}$$

The above equation gives precisely the magnon propagator of the usual spin-wave theory.

From (5.9), it can be seen that the longitudinal mode $\langle T[u\;u]\rangle$ has a pole on the imaginary axis. This nonphysical mode is related to the fact that there is no longitudinal dynamics in the lowest order of the spin-wave theory of the Heisenberg ferromagnetism. So, without losing physical information, we can also take $a_u = 0$.

By computing the propagator and vertices for the solution (3.5) at the same perturbative order, it is easy to show that the same results are obtained. In particular the free propagator takes the form (5.8) with $a = 0$.

In work under preparation, we will apply our perturbative approach to the renormalization and dumping of magnon energies.

6. CONCLUSIONS AND DISCUSSIONS

This paper has offered a new discussion of the path integral formalism for dynamical systems written in terms of Hubbard operators.

If it could have been possible, this path integral would have contained the full algebra (1.1)–(1.3) for the *X*-operators. Using the Faddeev–Jackiw symplectic formalism we showed that this proposal is not possible, and in order to satisfy the commutation rules (1.1) we cannot include the complete information contained in the *X*-operator algebra (1.1)–(1.3).

By consistency of the formalism and in order to satisfy the Hubbard commutation rules we found the number of constraint conditions. From our point of view and in a totally independent way we arrived at a path integral which is consistent with those obtained by means of the coherent states method.

We also showed that this path integral for the spin system case is valid in the large-spin-s limit. Then, we found that this limit is closely related to the impossibility of including the full algebra of the Hubbard *X*-operators in a classical dynamics.

On the basis of our path integral formulation we presented the diagrammatic and the Feynman rules for perturbation theory. We showed that our free theory is consistent with the results provided by the lowest order of the spin-wave theory.

Finally, we emphasize that from our approach a large family of kinetic terms of effective Lagrangians can be found, some of which can be related to previous Lagrangians obtained by different methods.

ACKNOWLEDGEMENTS

We acknowledge A. Dobry for many useful discussions. A.G. acknowledges parcial support on this project from the Fundación Antorchas.

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